*May 3, 2010*

*Learning, Arts, and The Brain*

Erice, Sicily, Italy 1-6 August 2010

**International School on Mind, Brain and Education**

Ettore Majorana Foundation and Centre for Scientific Culture

# PUT A BRAIN IN YOUR CAMERA: NON-STANDARD COMPUTER IMAGES

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This presentation is a story that started in November 27, 1987, when I delivered my inaugural lecture at The National Academy of Mathematics, Physics and Natural Sciences of Argentina about my implementation of a computer program for displaying three-dimensional objects or scenes in plane drawings following a non-standard method. It ends with a proposal to develop a proper machine using advanced high digital technology.

It also dealt with the geometrical description and computer generation of three-dimensional shapes. The importance of intrinsic geometry and modular techniques is analyzed as intricate forms can be created from simple building blocks. Using a three-dimensional system developed by the author architectural shapes can be reproduced and displayed from various viewpoints. The most important aspect of the system is its use as a rigorous and intuitive language for describing and generating objects, which may be applied also to robot guidance. Thus computers become versatile means of expression, providing people with opportunities to feel the emotion and joy of the creative act.

The main part of my presentation deals with a perspective system based upon curved projection rays. The method reduces certain distortions and rigidities observed by some people in classical perspective. The curvature of the rays is controlled by an image index, which can be selected at will by the user. Examples of images generated with different indexes are shown. The possibility of adding such an option to new visual devices, such as cameras or video, is raised, whereby the users can choose among alternatives according to taste. This opportunity to select the proper index would be possible thanks to modern techniques of computerized virtual reality and digital image processing. These ideas are rooted in the belief that it is essential to give people adequate resources for thinking and acting freely with machines.

## Introduction

Ever since the geometry of linear central perspective was developed in the 15th century, it has been observed that mechanical application of the procedure leads to effects of distortion and exaggeration of shapes and sizes, which often make the result look unnatural. Similar observations are made with the optical projections obtained in photography and video. Artists have intuitively corrected these perceptual shortcomings of perspective.
Before going into further, however, I believe it would be convenient to remember briefly the fundamentals of perspective.

The basic problem of representing the main properties of the shape of an object on a plane surface, is not simple. Strictly speaking, the visual concept of any object with volume can only be represented in a three-dimensional medium. But in trying to do this on a surface, we cannot expect to achieve anything better than the mere representation of some essential structural or particular factors of the visual concept. The drawings made this way may look plane, like a child's drawing, or may seem to have depth, like a picture with a vanishing point. But in both cases the total visual concept cannot be completely reproduced on a plane.

The representation on a plane of depth, or third dimension, has been a problem for artists and scientists in all times. From ancient rupestrian paintings to today's paintings, diverse modes of representation have been used.

In Egyptian drawings, heads and feet were always represented sideways. The artist wanted to show the essence of the objects, each part being drawn in a different position in order to emphasize its main characteristics.

Egyptian

In some ancient Oriental paintings, parallel lines from reality remain parallel in the drawing without any vanishing point, according to the laws of axonometric (or parallel) projection.

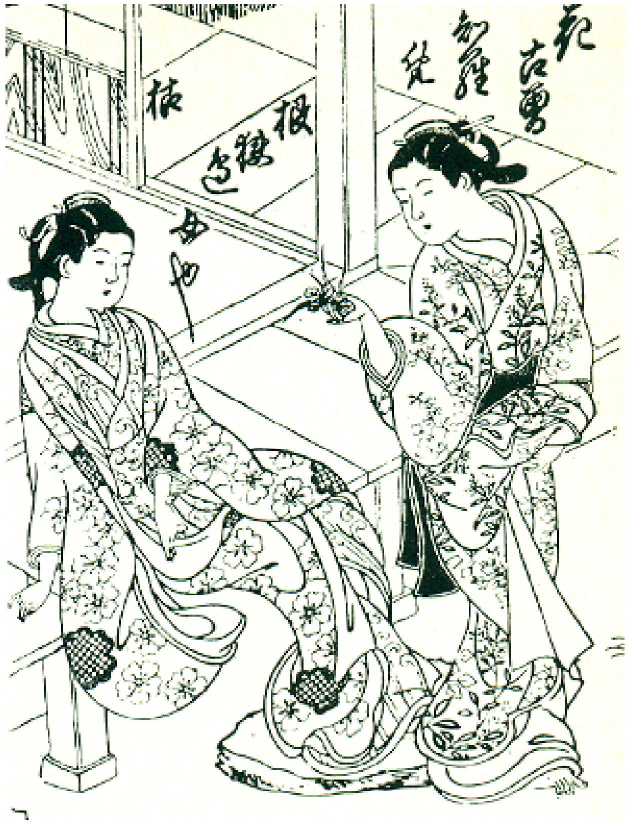
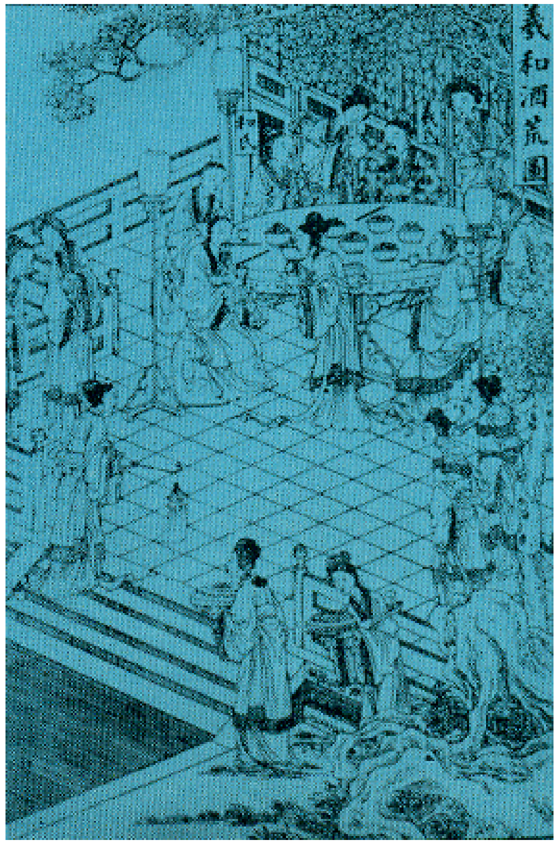


Figure Japan, XVIII

In other pictures, the elements are drawn separately, at different superimposed planes and in different scales, as it was seen in old Greek pictures and in some Pre- Renaissance paintings.



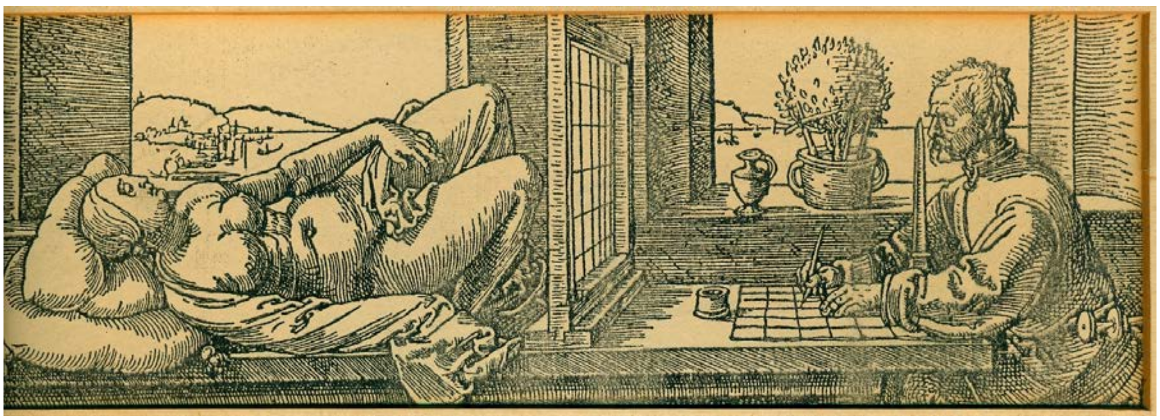
Axonometric (or parallel) projection

During the XVth century the use of perspective based on the rules of conical projection spread in the West. Although some simple perspective was already used in Euclides's Greece, its fundamentals were scientifically defined and its use became widely accepted during the Renaissance.

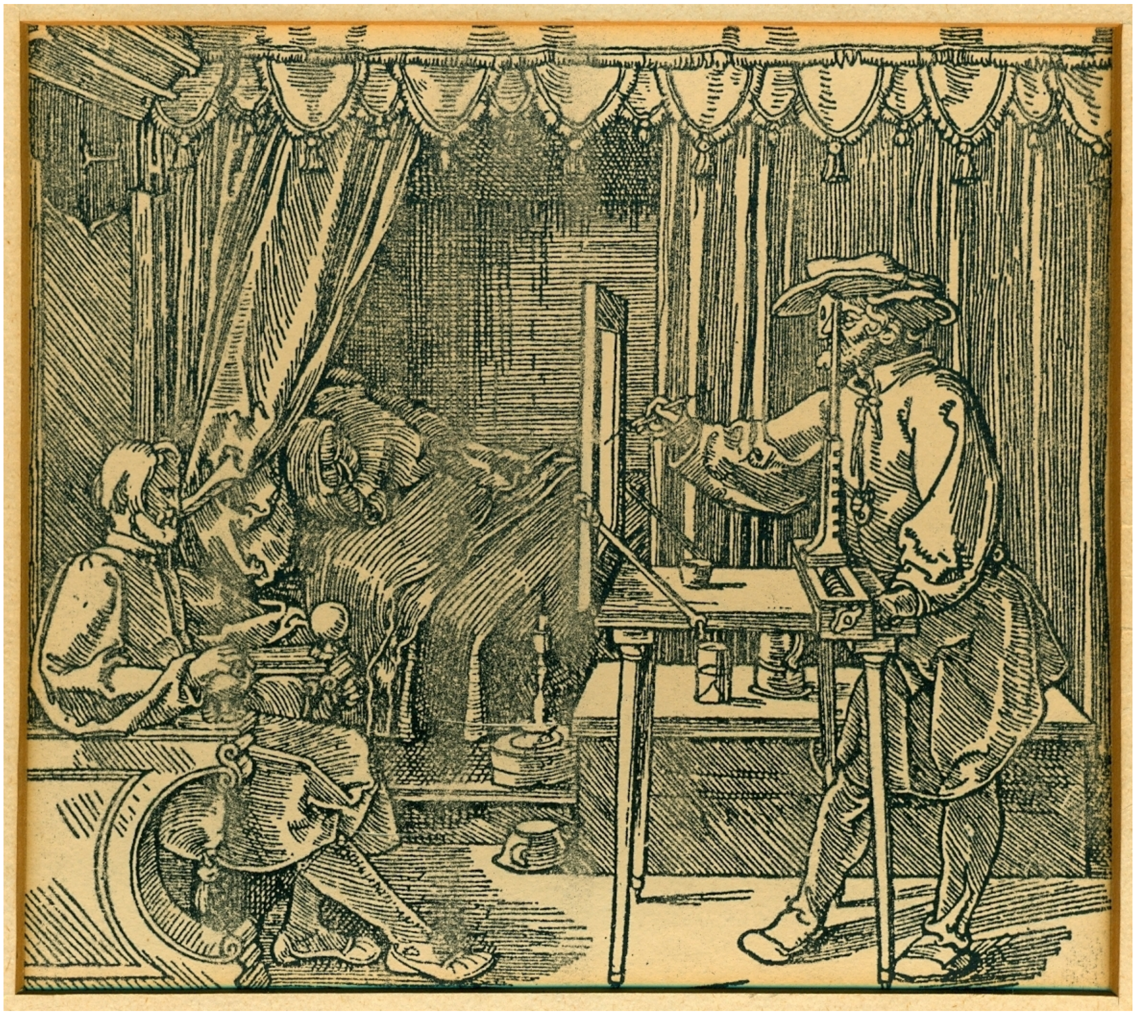
It was Leonardo da Vinci (1452-1519) who clearly explained the representation of objects in perspective. He did it in this way: let us suppose an observer who sees an object through a glass window, and paints on the glass the intersection points of straight rays to the points of the object, in such a way that the successive opaque colour points, painted on the glass, cover the view of the corresponding points of the object.
In the usual three-dimensional systems utilized, Leonardo's window glass coincides with the computer screen, and the

observer's eye or point of view -the centre of the projection- with a fixed point situated in front of the centre of the screen.

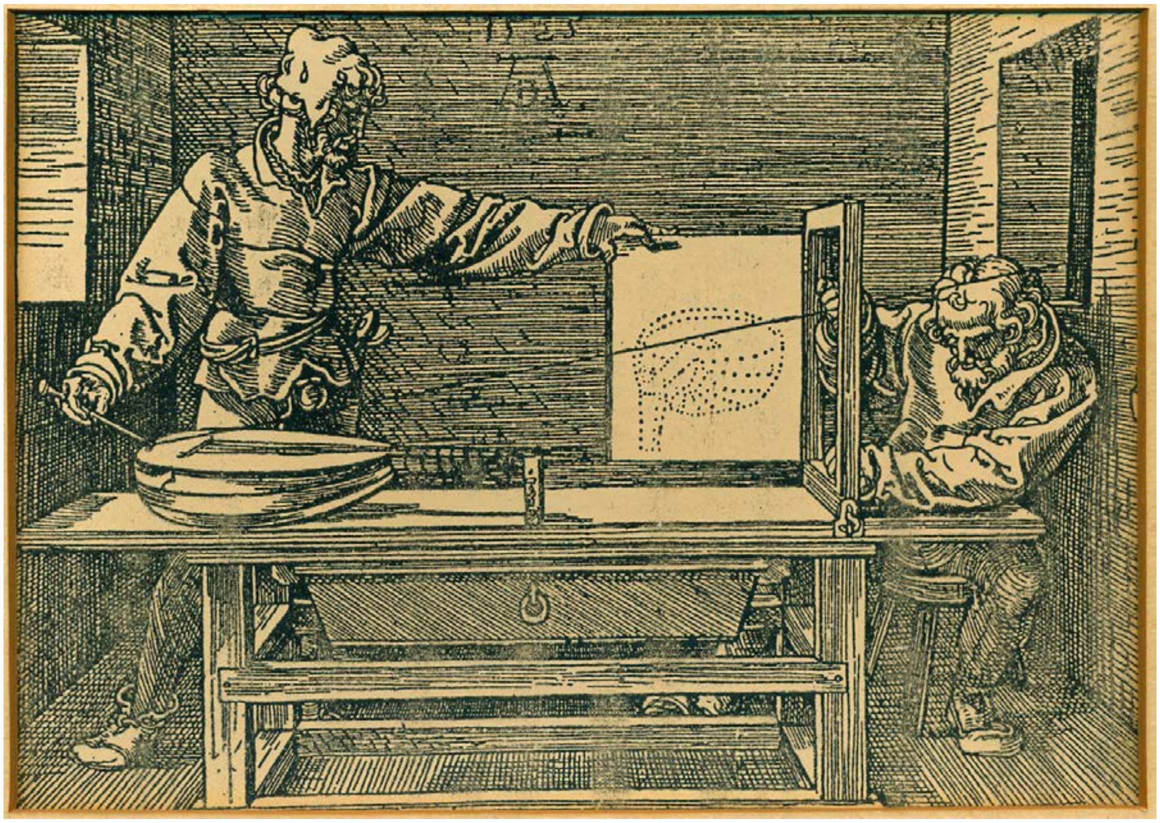
Albrecht Dürer (1471-1528), around 1523, made a set of woodcuttings showing several ingenious optical-mechanical devices developed to obtain conical perspectives of objects or scenes. Is it interesting to point out that the usual computer procedures exactly reproduce Dürer's machines in the mathematical terms of the computer.



Dürer 1

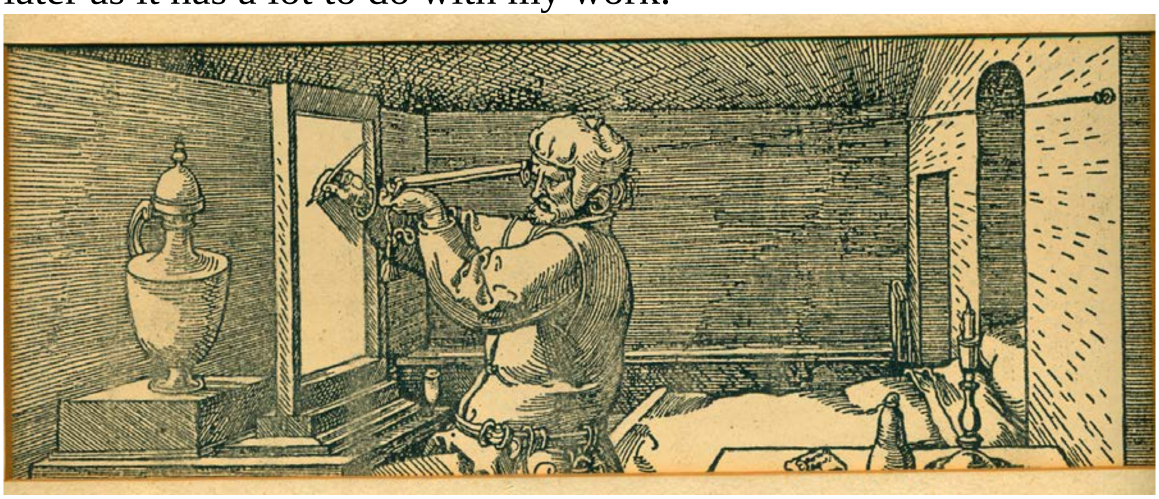


Dürer 2



Dürer 3

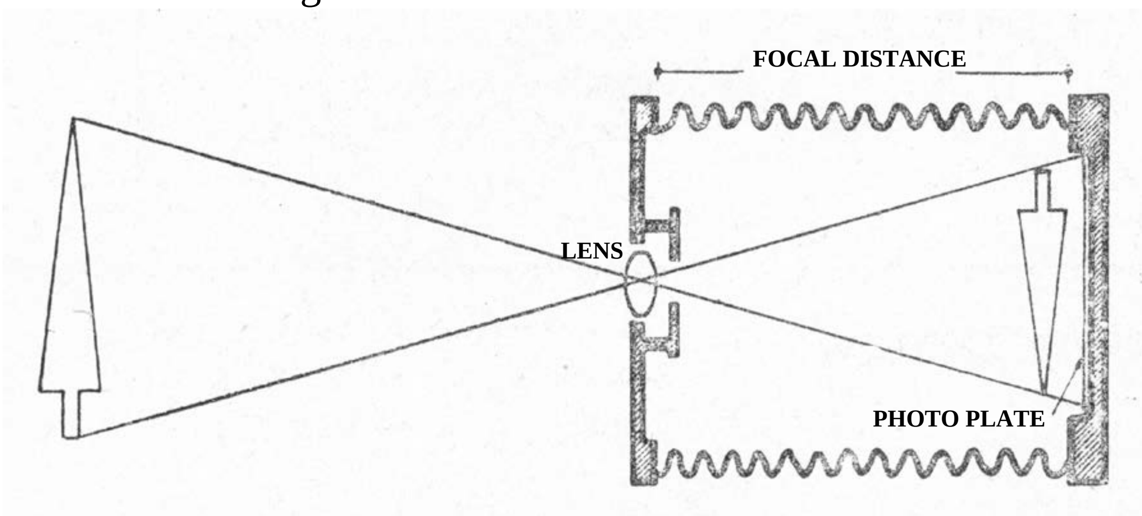
There is a notable fourth Dürer's woodcutting to which I will return later as it has a lot to do with my work.



Dürer 4

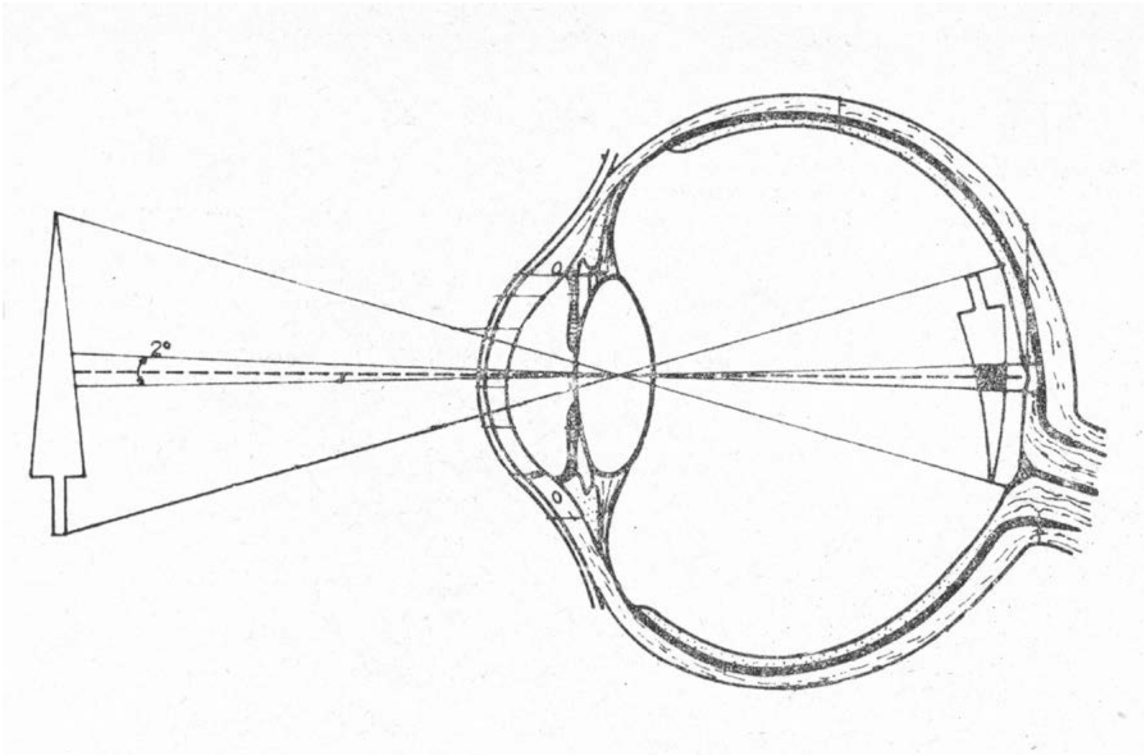
Woodcut by Albrecht Dürer showing the construction of a drawing in linear perspective with the viewpoint behind the artist.

The photographic cameras invented by N. Niepce (1765-1833) and J. Daguerre (1757-1851) also reproduce Dürer's ideas, this time with lenses and light-sensitive film.



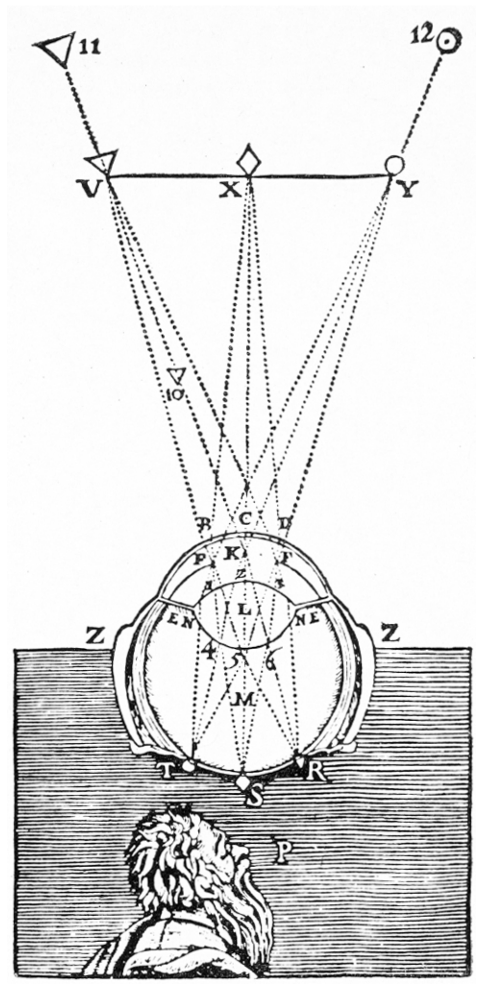

Niepce – Daguerre

It is worthwhile to mention that this perspective representation is also what occurs when the image of an object is generated in the human eye. The cornea and the lens make up an optical system that produces images on the retina of the light rays coming from the objects in front.




Eye

It is interesting to remember that René Descartes (1596-1650), author of *La Dioptrique* (Leyden, 1637), put the eye of an ox on a hole of a table. Looking later from the back of the eye that he has scraped for making it transparent, he could see a small inverted image of was at the other side of the table. In some later years, the human eye was compared with a photographic camera. It was quite bad, that the Descartes illustration suggested the wrong idea that the person who sees something, is seeing from behind his retina the figures projected on it.



From Descartes Dioptrique

All this has frequently led to comparisons between the intricate process of visual perception and perspective. It must be understood, however, that conical perspective laws are not the laws of "how we see"; they are only a simple and useful method for plane representation. Visual perception is not exclusively a process of capturing sensorial information: it is, moreover, an exploration and search process, where sensorial information is used in the comparison, construction and selection of diverse prior perceptual hypotheses that are interactively contrasted with the present hypotheses. The perceptual act cannot be completely represented on a plane.

Descartes himself clearly wrote in *La Dioptrique*: “I need not say anything special about the way we see the size and shape of objects; it is completely determined by the way we see the distance and position of their parts. Thus, their size is judged according to our knowledge or opinion as to their distance, in conjunction with the size of the images that they impress on the back of the eyes. It is not the absolute size of the images that counts. Clearly they are a hundred times bigger when the objects are very close to us than when are ten times farther away; but they do not make us see the objects a hundred times bigger (the area, not the linear size); on the contrary, they seem almost the same size, at any rate so long as we are not deceived by (too great) a distance”.

On his "School of Athens", 1509, for example, Raphael (1483-1520) made the shapes of Plato and Aristotle larger than they should have been according to the strict conical perspective based on the architectural lines.

"School of Athens"-Raphael

The same thing happens in the painting "Dinner at Levi's House", painted, in 1573, by Paul Veronese (1528-1588).

"Dinner at Levi's House"-Veronese

The figure of Christ is larger than the size corresponding to the perspective used in the architectural setting; if Christ stood up, even though he is in the background, he would be as tall as the figures in front. It is also interesting to cite Pierre Renoir's comment -transcribed by Ambroise Vollard in his book "La vie et l'oeuvre de Pierre-August Renoir"- referring to another famous Veronese picture, "Cana's Wedding".

"Cana's Wedding"-Veronese

Ambroise Vollard wrote: "If the picture were in real perspective, it would be empty, because the figures in the background, which are as large as those in front and would be very small. Nor does the floor follow the rules of the lines of perspective. Perhaps for all these reasons the picture is so beautiful."

Painters through all time have corrected, in some way or another, the perceptual deficiencies of rigorous conical perspective. I attempted something similar in 1974, when I developed a slightly different method based upon a sort of curved projection rays. I intended to diminish the distortions and to obtain images that looked more natural and acceptable, according to each personal taste.

## How to describe and generate three-dimensional shapes

Now I shall refer to two subjects I have done research and development over several years. The first one is how to describe and generate three-dimensional shapes. We all know the objects around us play specific roles, and so must keep their shapes through time, resisting the forces that act upon them. I shall deal not with the structural or resistant aspects of the forms, but only with their geometrical aspects.

Then I will deal with a second subject: the representation of three-dimensional objects on a plane, that is, the problem of drawing the shape of a spatial object on a surface.

I have elaborated my ideas on both subjects in computer programs. I believe that computers have the potential of becoming a useful medium of expression and that everybody would be able to communicate with them in a creative and personal way. I am also convinced that, more important than the multiple applications of computers, are the new ideas inherent in their essence and in the proper choice of how they are used.

**(It continues in Appendix3D)**

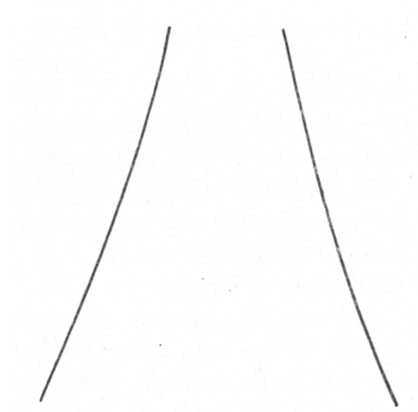
## The representation of three-dimensional shapes

The problem of the 2-D representation of 3-D shapes as done by conical, parallel or other methods, must only be considered as partial models of visual space.
When classical perspective became common-place in the world of painting, many artists found that exact application of its rules led in

some cases to exaggerated and undesirable distortions in shapes and sizes of the resulting drawings.

It is known that in classical conical perspective, dimensions of images change in inverse proportion to the distance of the objects from the viewpoint. This also applies to images produced in the retina of the eye, as well as in photographic film. But our subjective estimation of the sizes of objects does not exactly obey that law. A simple way to prove this is by looking at a photo of a group of people taken very close to them: the images of the faces of those who are more distant appear much smaller than those who are closer, even smaller than we perceived them while photographing. Photographs of mountains frequently surprise us: they look smaller on film than in reality. The sharp convergence of the sides of a skyscraper, photographed with a camera pointing upwards, draws our attention: we see that the lines converge to a point when we look at a tall object from below, but this convergence does not seem as sharp as it appears in the photograph.

Another observation is the presence of curved lines in some paintings, as well as in Greek temples; small deformations of the straight lines improved the beauty of the proportions. Two centuries ago Robert Smith (1738) said that the parallel sides of a very long and broad avenue, marked with vertical elements, do not appear to converge like two straight lines but rather like two curved lines meeting asymptotically, like this:



SMITH CURVES

Another illustrative visual experiment arises when an individual, located in a fixed position, guides the placing of pairs of poles equally separated, at progressively greater distances. The poles so placed define curves called "Hillebrand avenues", of curvature contrary to Smith's curves. These curves are like this:

HILLEBRAND AVENUES

All these circumstances are, in principle, explained by the psychological phenomenum of size constancy. This is the name of the supposed tendency of visual perception to compensate for changes in retinal images with different viewing distances. Essentially, distant objects are enlarged; near images are diminished.

The English psychologist Robert Thouless from Cambridge University carried out the first significant experiments about the importance of size constancy –called also size consistency-. Thouless called it "phenomenal regression to the real object" and thought it convenient to define a "regression index" to measure this tendency:

$$i = (\log p - \log s)/(\log r - \log s)$$

where the symbols stand for:

p: apparent or phenomenal size of the object.
s: theoretical or stimulus size corresponding to the given distance.
r: real size of the object.

THOULESS FORMULA

According to the formula, a zero value of the regression index would indicate the absence of phenomenological regression, that is, the phenomenal size would coincide with the stimulus, while a unity index would indicate a total regression, that is, coincidence with the real size.

My idea was to use Thouless's numerical measure formula as a basis for an equation that relates variable values of $p$, $s$ and $r$ for a fixed value of index $i$, deducing value p from his formula:

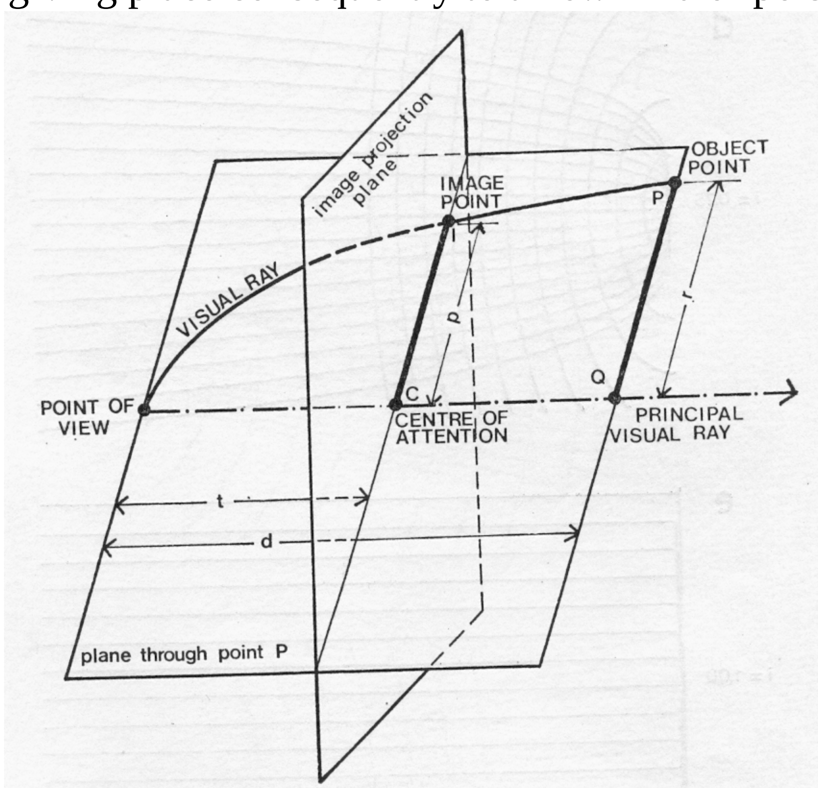
$$p = r \cdot (s / r)^{1-i}$$

Knowing that the stimulus size s, at a distance d, with regards to real size r, is $s = r \cdot (t / d)$, results:

$$p = r \cdot (t / d)^{1-i}$$

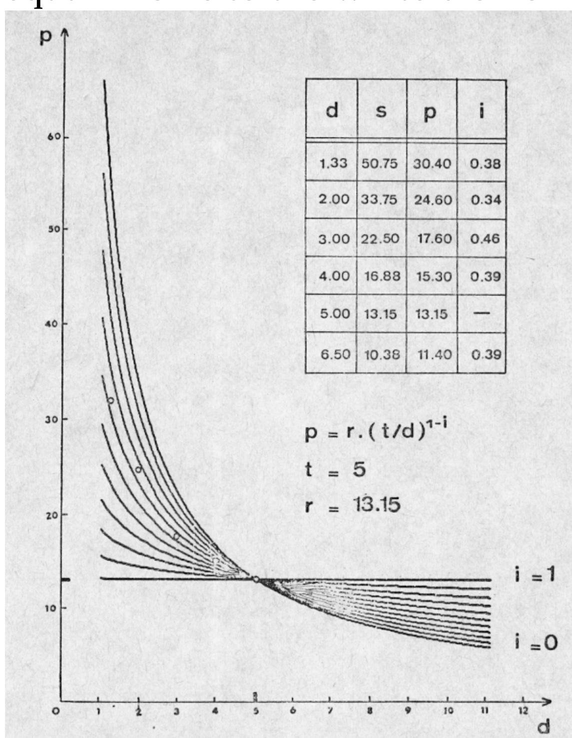
CURVED RAY EQUATION

This last equation may be interpreted as a relation between a real point of the object of coordinates ( d, r ) and its corresponding image point of coordinates ( t, p ). This reasoning led me to accept a curved projection ray with a geometry defined by the last equation, giving place consequently to a new kind of perspective.



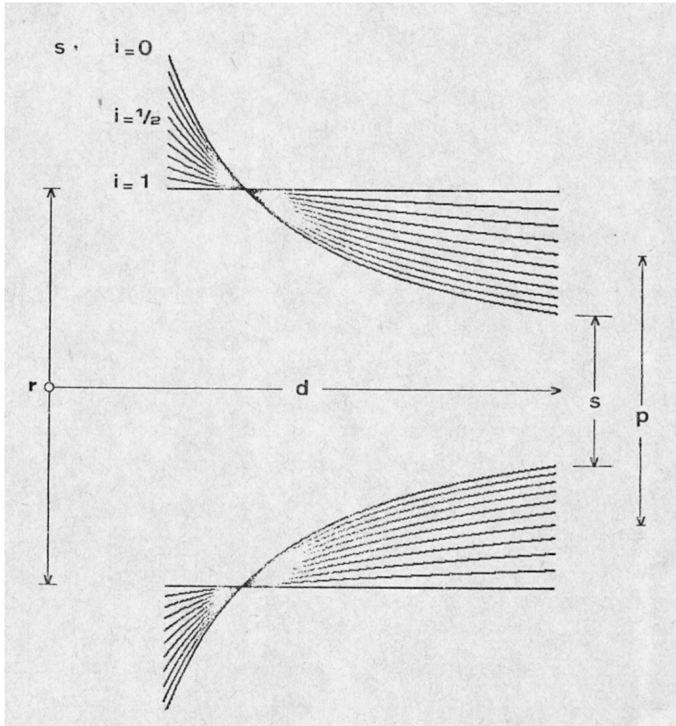

PROJECTION OF A CURVED RAY
A GENERALIZED PERSPECTIVE GENERATION

Thouless performed a series of experiments in Africa and determined with his formula the index i of many people, always finding values between 0 and 1. In one of his experiments he projected through a lantern with a variable diaphragm a small circle upon a screen at 5 m distance. At intervals, he vertically showed to a subject a white circular disk of 13,15 cm, at successive distances 1.33, 2, 3, 4, 5 and 6.50. For each of those positions, he asked to the subject to regulate the opening of the diaphragm in such a way that the size of the circle projected upon the screen, appeared to him equal in size to the white disk circle.



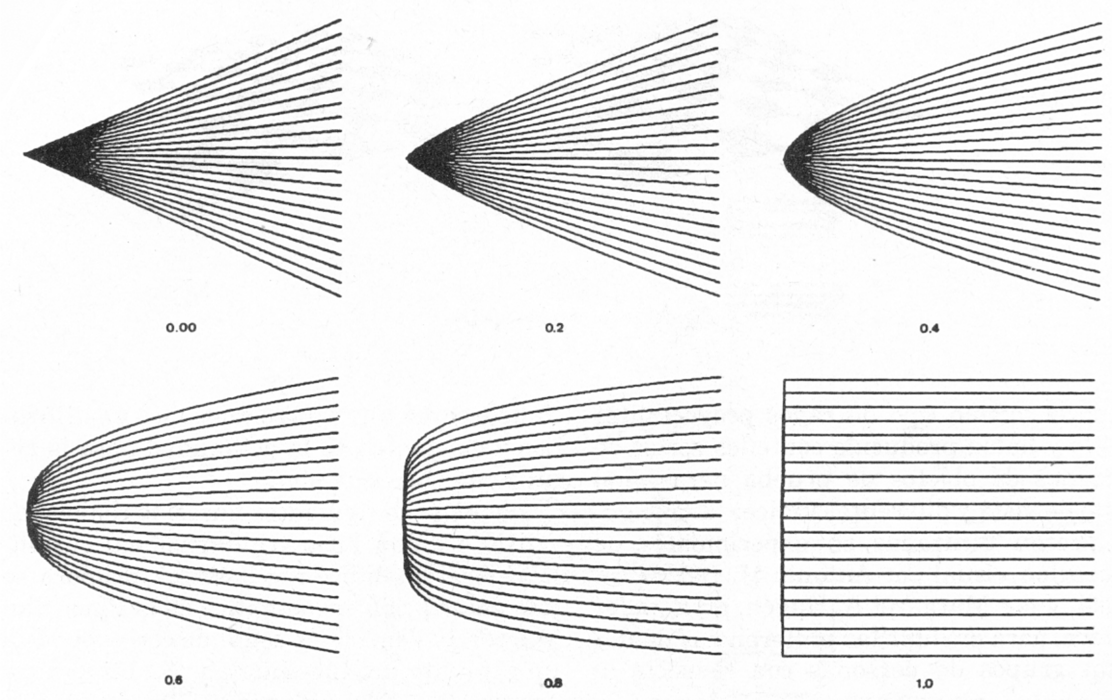

| d | s | p | i |
|---|---|---|---|
| 1.33 | 50.75 | 30.40 | 0.38 |
| 2.00 | 33.75 | 24.60 | 0.34 |
| 3.00 | 22.50 | 17.60 | 0.46 |
| 4.00 | 16.88 | 15.30 | 0.39 |
| 5.00 | 13.15 | 13.15 | — |
| 6.50 | 10.38 | 11.40 | 0.39 |

THOULESS EXPERIMENT
CURVES FOR DIFFERENT INDEXES SIMILAR
TO SMITH CURVES



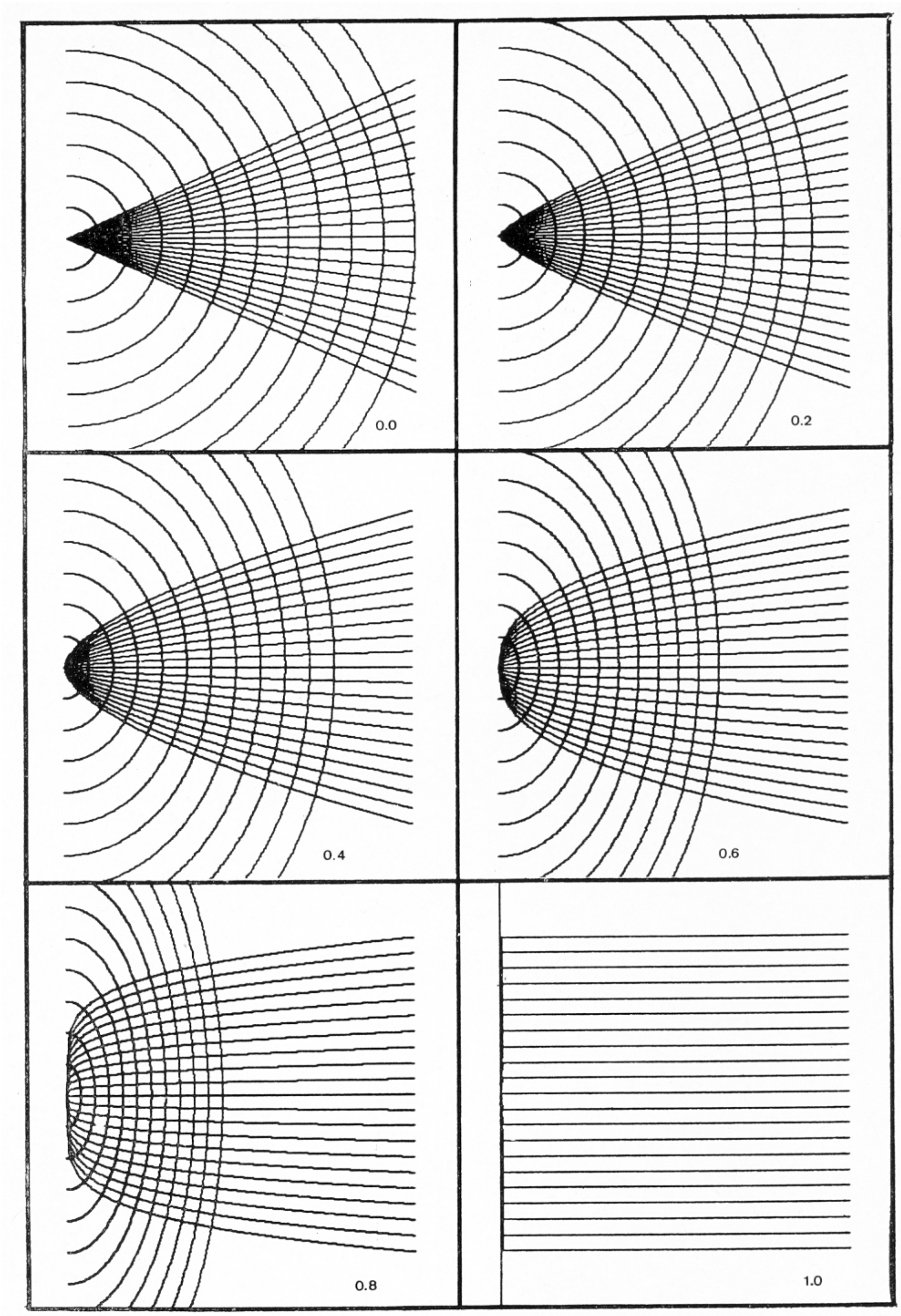

THOULESS EXPERIMENT
CURVES FOR DIFFERENT INDEXES SIMILAR
TO HILLEBRAND AVENUES

The following figures show families of curved projection rays for different values of i:




CURVED RAY FAMILIES
FOR DIFFERENT INDEX VALUES

The orthogonal trajectories to the curves projection rays –known mathematically as “eikonal lines”- are shown below:




EIKONAL LINE FAMILIES
FOR DIFFERENT INDEX VALUES

Those eikonal lines have been obtained solving the differential equation:

$$y' = -(x/y) \cdot (1 - i)$$

which results replacing $y'$ by $-1/y'$ in the differential equation of projection rays:

$$y' = (y/x) / (1 - i)$$

where $y$ is $p$ and $x$ is $t$.

The eikonal lines are elipses with center at the origin and semiaxises along x and y axis in the relation $(1 - i)^{1/2}$ :

$$x^2 / (g^2 \cdot (1-i)) + y^2 / g^2 = 1$$

where $g$ is a parameter.

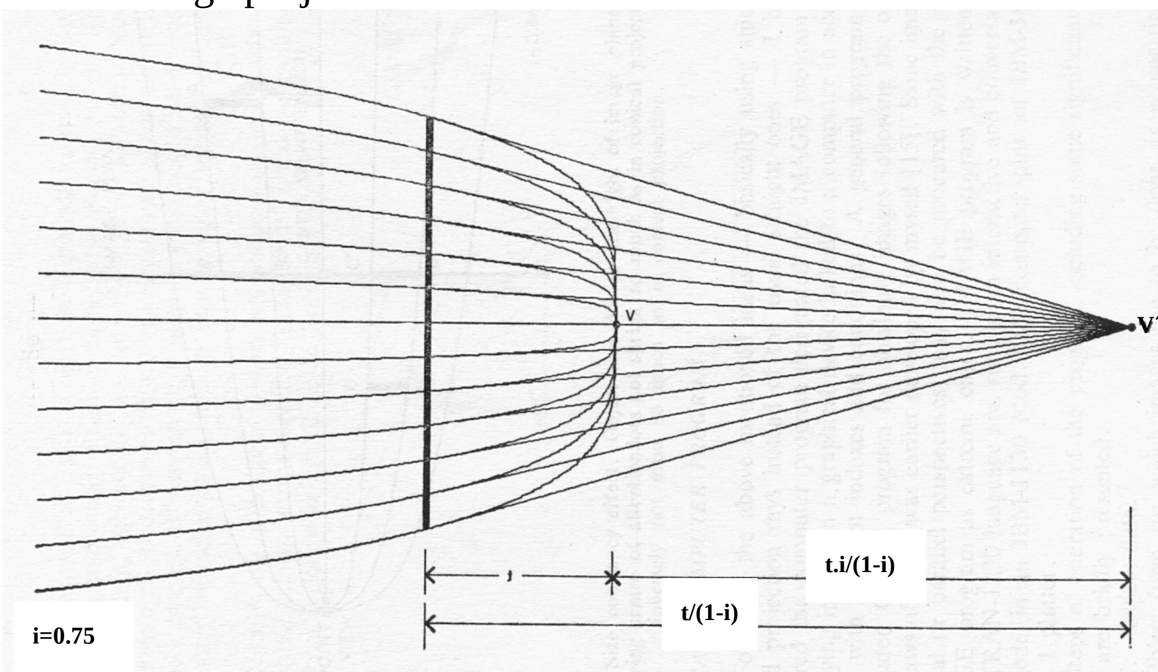
We could think that those eikonal lines are as fantastic layers of transparent material that act as refracting media with index of refraction i for the supposed curved rays.

As previously explained, the projection rays used in the proposed generalized perspective create an image in the hypothesis of invariant index i for different sizes and distances. Antonio M. Battro, in Brazil, and Margaret A. Hagen, in U.S.A., has done some experimental evidence that supports the hypothesis of invariant of index i in a wide range of distances.

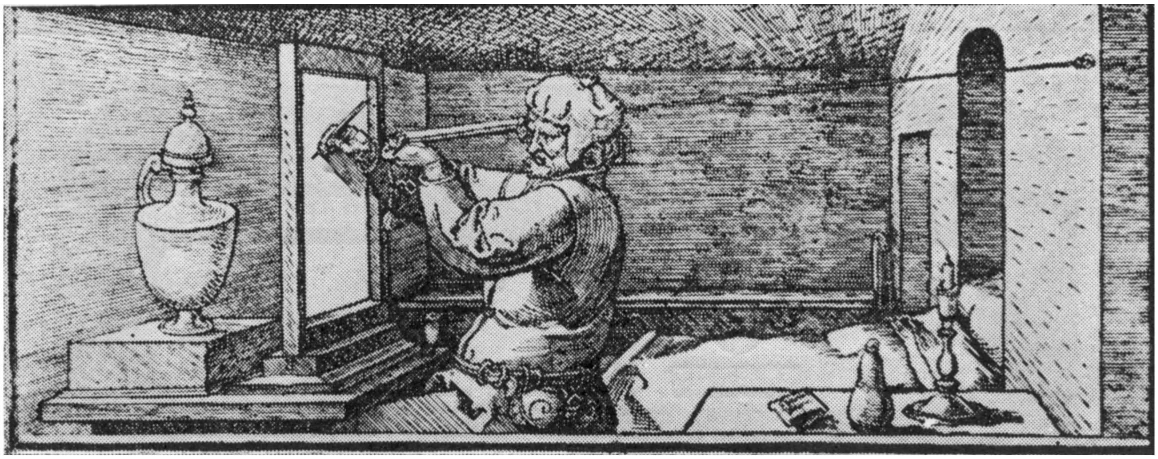
I consider worthwhile to comment some characteristics and properties of the curved rays:

- The principal straight projection ray concentrically supports a bundle of spatial curved projection rays. Thus, the generated image is a cross section of said shape of revolution around the principal straight projection ray.
- Tangent line slope decreases from infinity at the vertex to zero at far distance.

- All tangent lines to the projection rays at a same distance from the origin concur to a unique point behind the vertex. This property explains in a sense the resemblance between an image generated with the proposed curved rays and the image based upon traditional straight rays departing from a further point of view. Such a point may be regarded as a pseudo-point-of-view for the curved image projection.

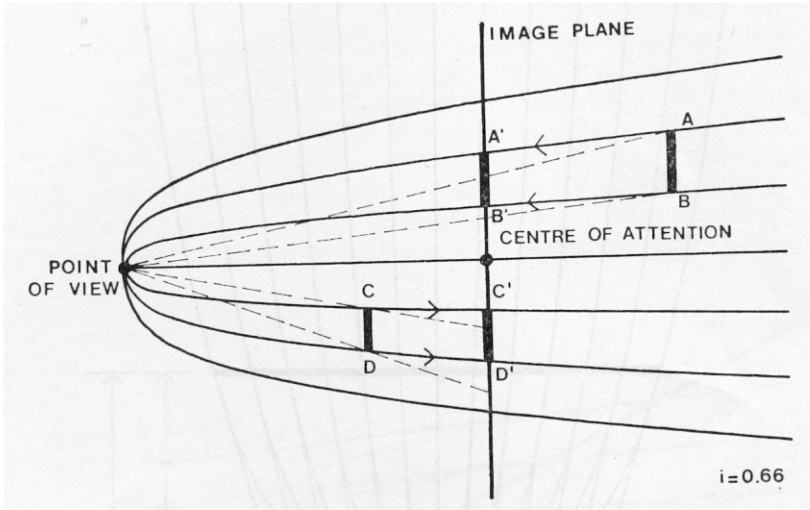



A technique for making a drawing in linear perspective with the viewpoint behing the artist was depicted by Albrecht Dürer in a one of his woodcut as was already shown. The artist sights along a string that is attached to a wall by a pin which represents the viewpoint.


Dürer 4

Dürer engraving which shows a painter drawing according the laws of conical perspective. The point of view is the pin wall to which one string extreme is fixed. In this manner the resulting picture does not geometrically agree with the painter position but with one behind him. In odd analogy, the generalized proposed perspective also produces drawing resembling pictures taken from a farther viewpoint than the actual one. They are very similar when the distance between real point of view and the centre of attention is not small.

- An object entirely contained in a normal plane to the principal ray casts a geometrical similar figure upon the image projection plane. Thus, it is reproduced without distortion giving same figure in other scale.
- As a corollary of former property, it is deduced that images obtained in different projection planes (that is, located at different distances from the vertex) are geometrical similar.
- Assumed curved projection rays in comparison with straight ones make smaller the close objects and bigger the far ones, thus geometrically introduces a kind of size constancy effect.



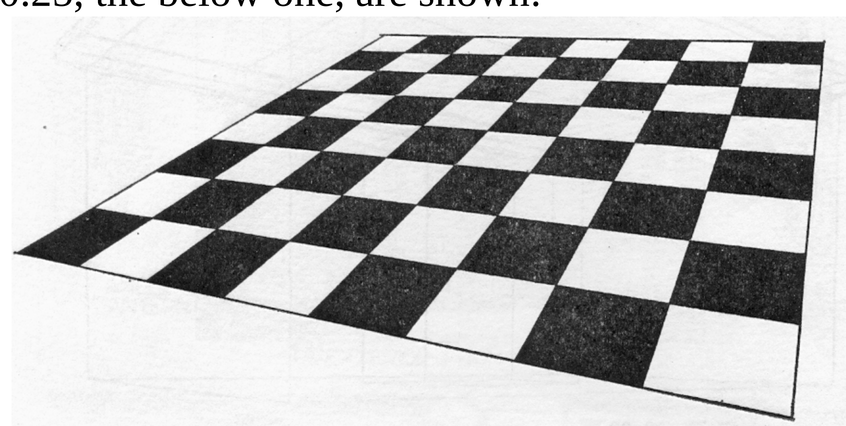

Size constancy has been a controversial issue and has been discussed by many researchers in the past, among other by Rudolf Arnheim and Richard Gregory. Although it has been incompletely understood how it is mediated, it has been accepted that at least two factors are relevant: the present of contextual stimuli simultaneously present in the visual field
and experience observing such stimuli. H. W. Lewibowittz stated that perceptual constancy illustrates well the circumstance that the organism itself contributes significantly to the process of perception and expressed his opinion that perceptual constancy is of critical importance to our understanding of living organisms.
E. H. Gombrich wrote about size constancy: “Our expectation that a small object in the distance would prove to be larger than it appears to be at the moment, once we approach it, notoriously make us see distant objects as larger than their retinal size would allow us to infer. This is the so-called constancy phenomenon. The

term has been criticized because, as Dr. Thouless has stressed, phenomenal size appears to be a compromise between retinal size and inferred size. Personally I am not very happy with the concept of phenomenal size altogether, because in real life situation it proves to be a very elusive entity”.
Gregory commented in his latest works about constancy-scaling in the following way: “there was a long-standing muddle about apparent size and shape being somehow compromises between retinal images and reality; this is a hangover from the direct, or intuitionist, view of perception and is still to be found. In my view, it is metaphysical and deeply confusing”.

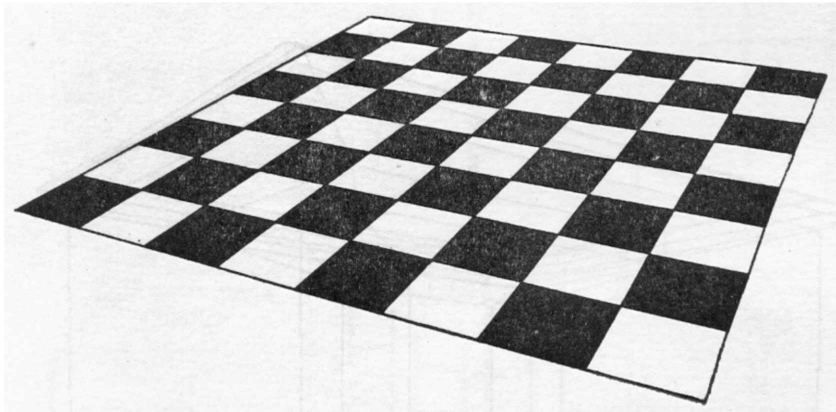
# Examples

Now, I want to show some pictures which were generated using the method I have explained.
The figures generated with the new perspective range from conical to parallel. With $i = 0$, the classical conical perspective is achieved; with $i = 1$, the classical parallel perspective. For a given object, the curved rays enlarge the frontal dimensions of the parts distant from the viewpoint, and shrink the near dimensions, compared with the conical perspective of straight projection rays.

**Checkboard**
Two images of a checkboard with indexes $i = 0$, the above one, and $I = 0.25$, the below one, are shown:



$i = 0$

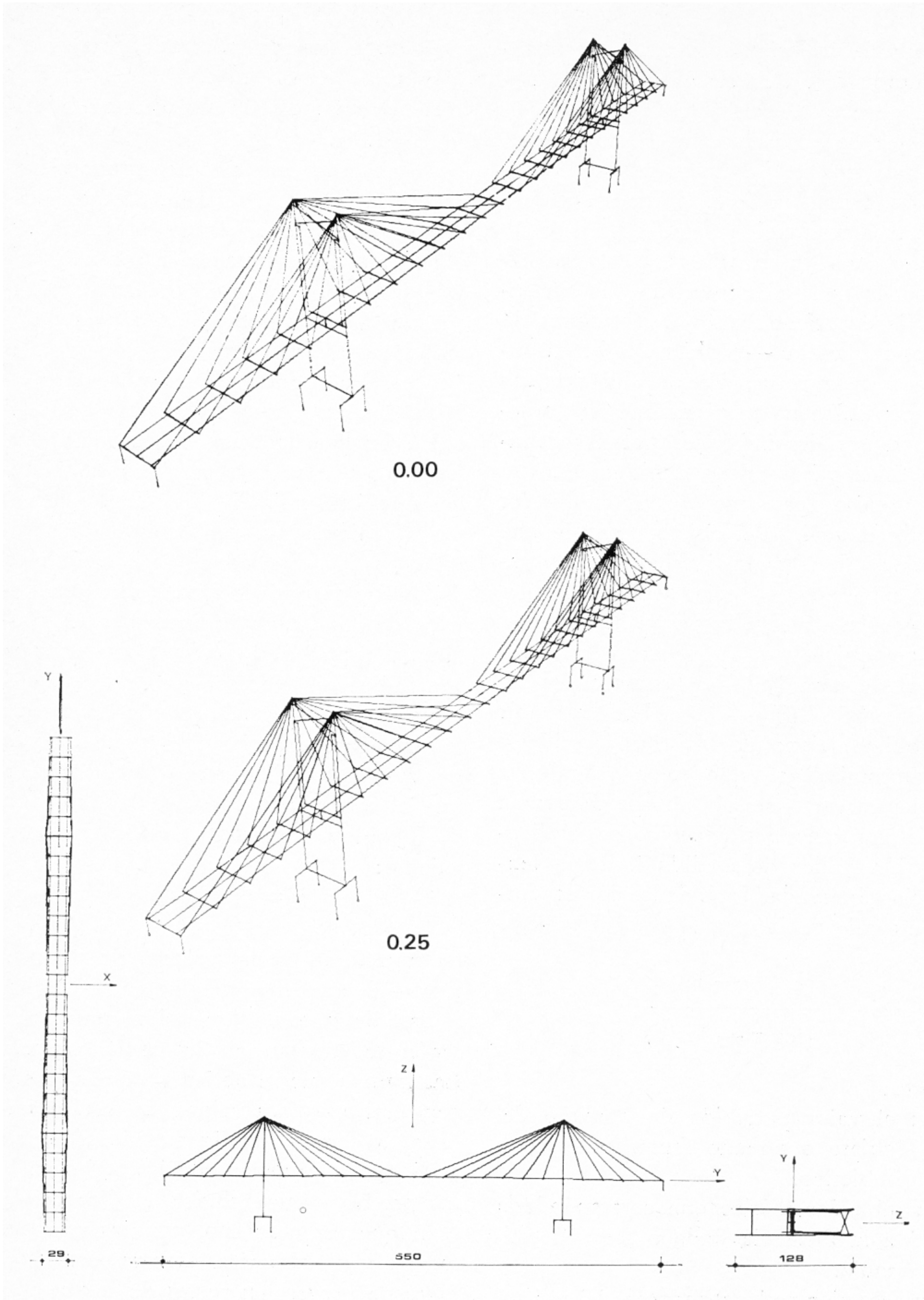


I = 0.25

It is worthwhile to note that the straight lines of the board are generally transformed by the new perspective into curved lines. Curvature of lines is only noticeable in those cases where the viewpoint and the object are very close and is almost imperceptible with normal distances.

**Zarate-BrazoLargo Bridge of Argentina:**

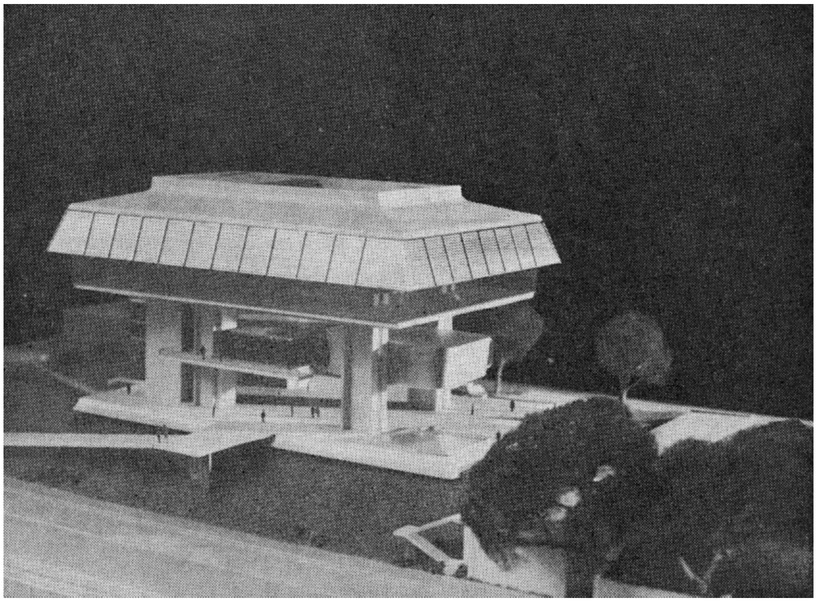
Idem, of a structural model of the Zarate-BrazoLargo Bridge of Argentina:

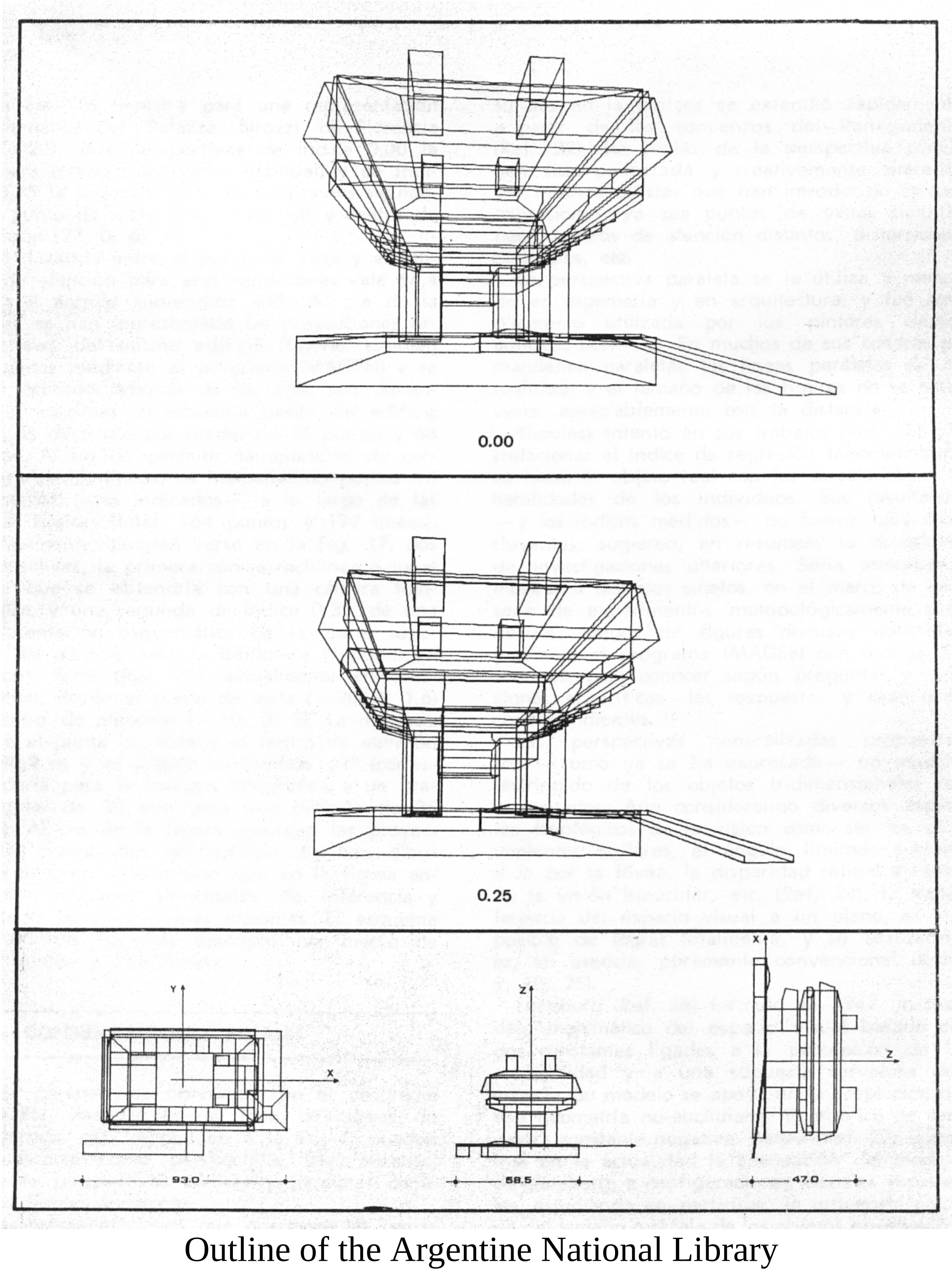



Both images are taken from far away. The two images are taken from viewpoint (250; -400; 250) and centre of attention (0; 0; 22). The convergence or vanishing of the lines corresponding to parallel lines of the bridge is not as noticeable as in classical conical perspective of zero index.

**Argentine National Library**:

Photograph of a maquette of the Argentine National Library:



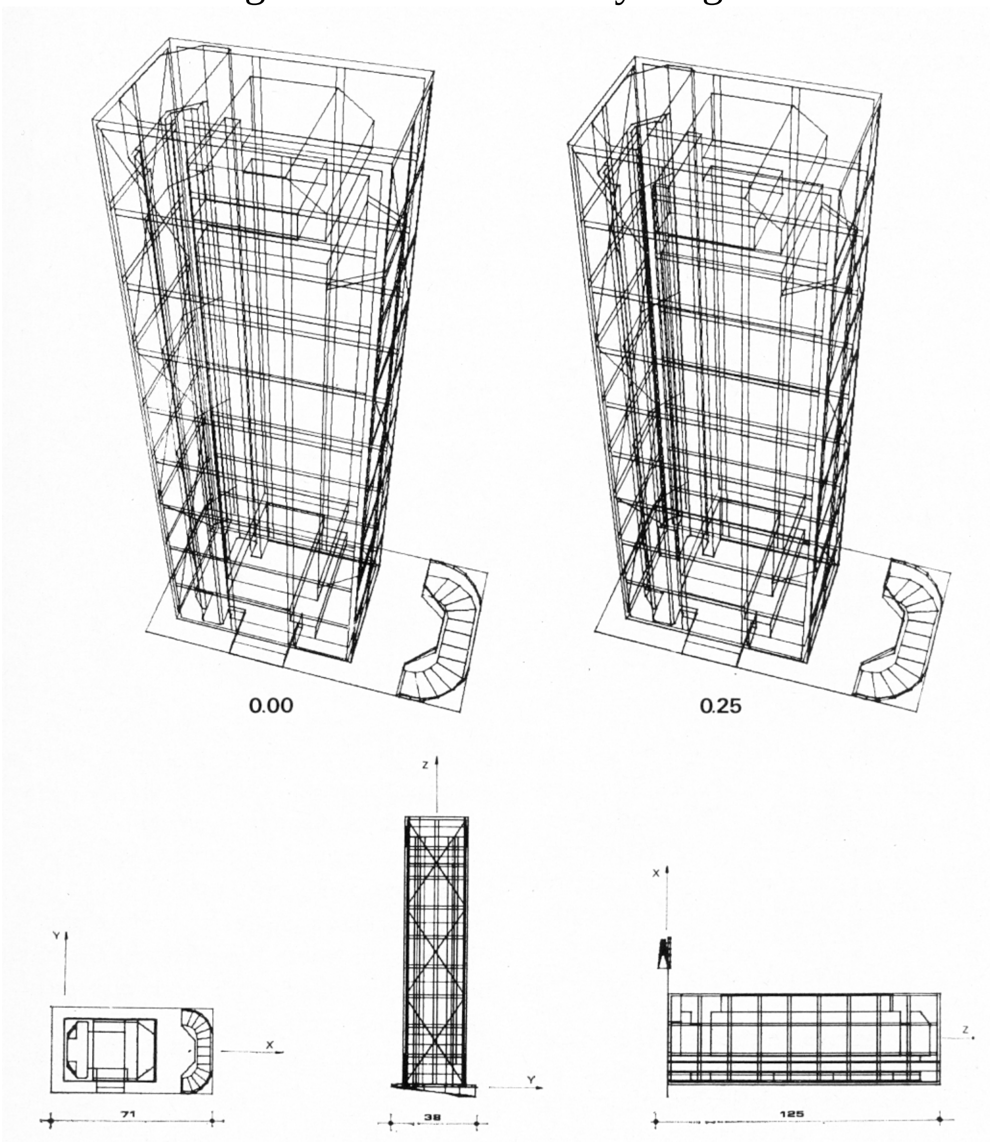



Outline of the Argentine National Library

The two images are taken from viewpoint (-90; 6; 1.60) and centre of attention (-10; 0; 2).

**A Building in Buenos Aires**:

Model of a building in Buenos Aires City. Argentina:

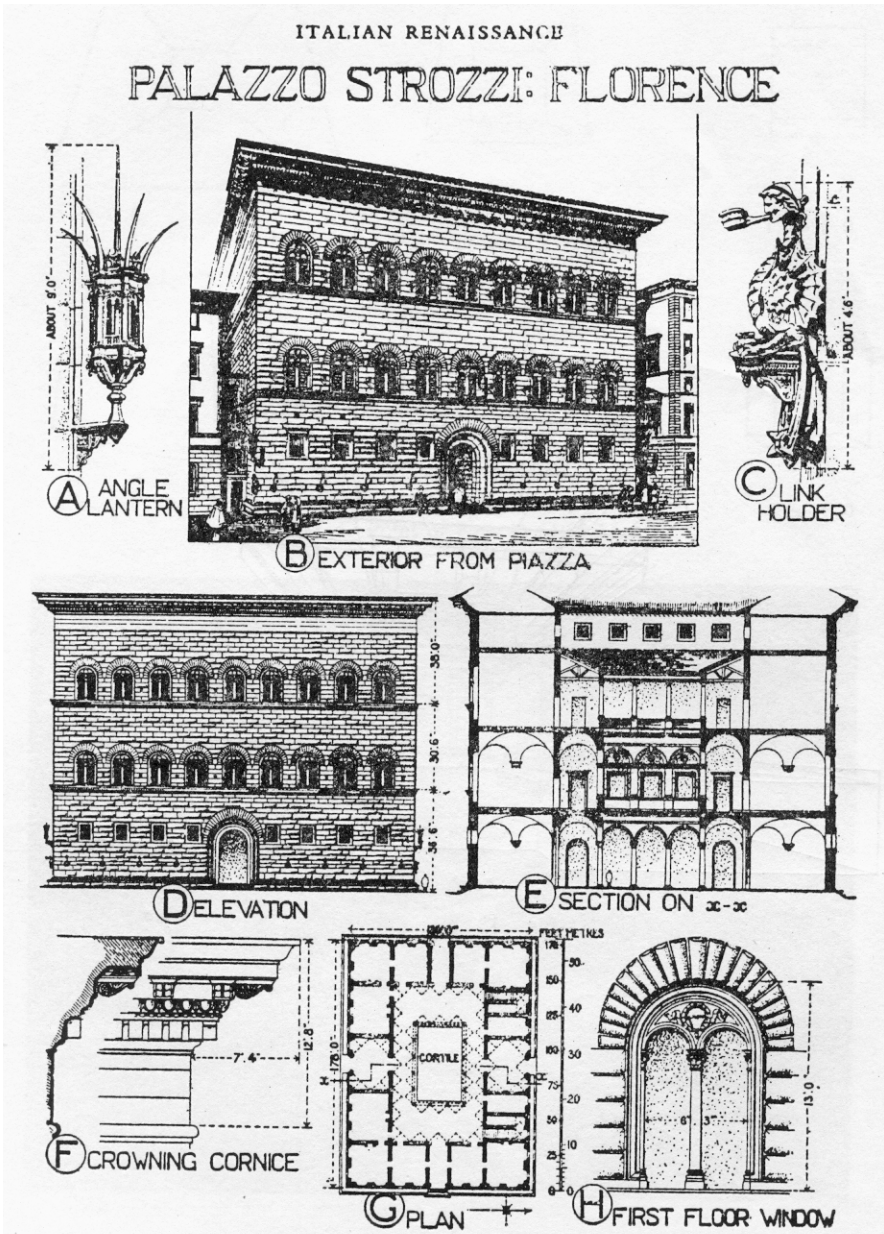



Outline of a building

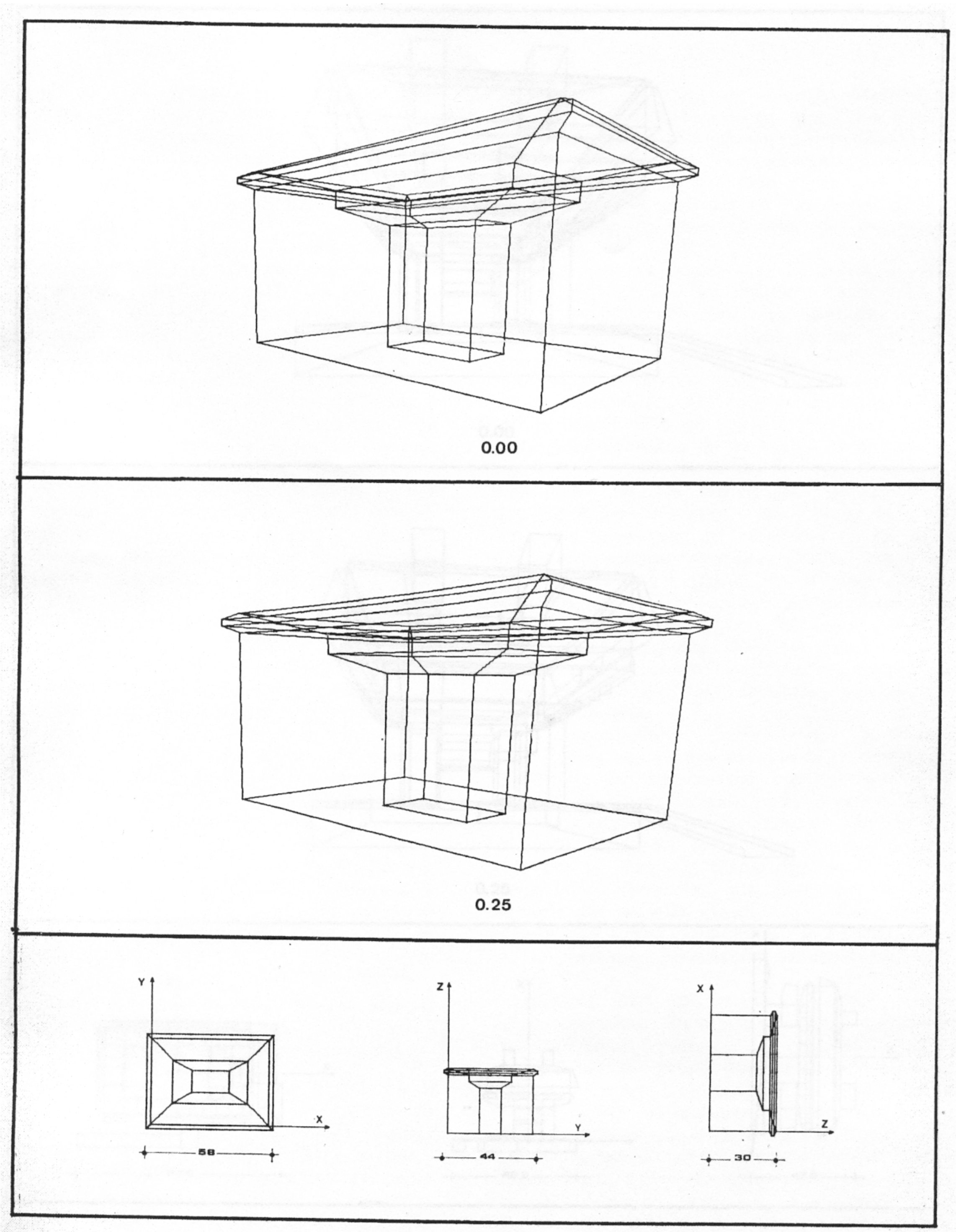
The two images are taken from viewpoint (60; -120; 180) and centre of attention (25; 0; 46).
The convergence or vanishing of the lines corresponding to vertical lines of the building is not as noticeable as in classical conical perspective.

**Palazzo Strozzi**

The Palazzo Strozzi of Florence, Italian Renaissance:




Palazzo Strozzi

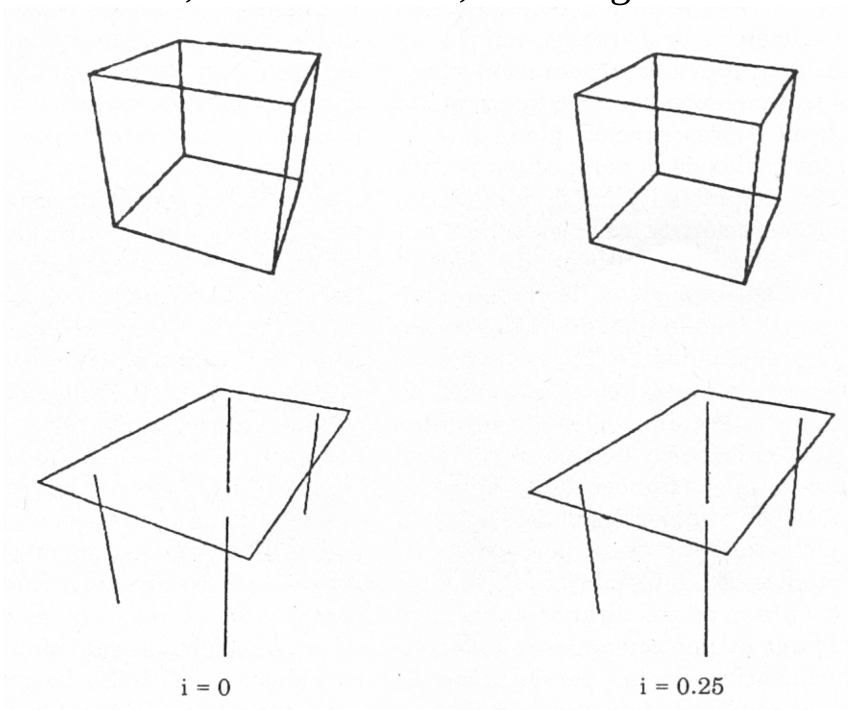



Outline of Palazzo Strozzi

The two images are taken from viewpoint (75; -45; 15) and centre of attention (27; 0; 6).

**Cube and Table**

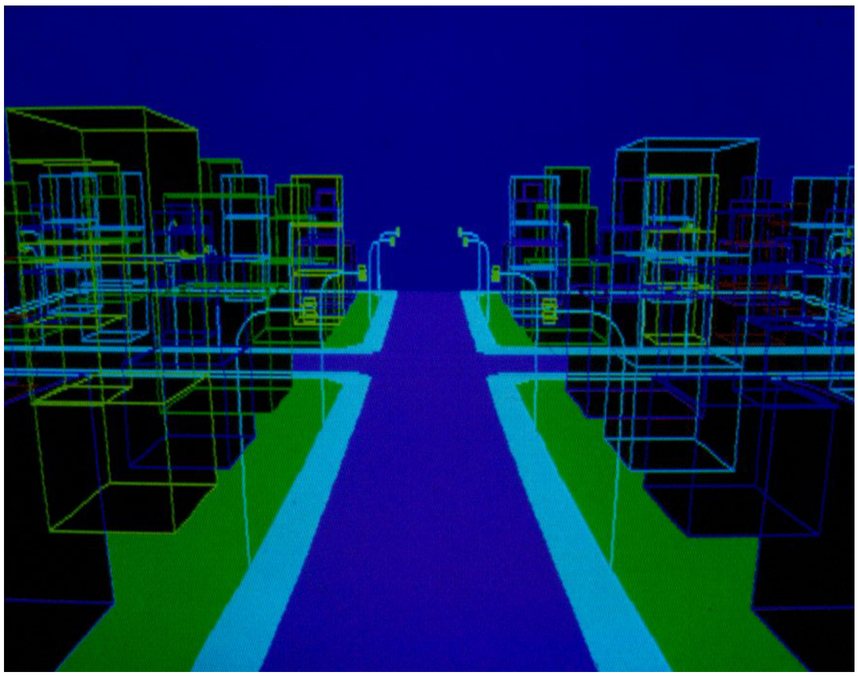
A Cube and an outline of a Table from a close viewpoint with index 0, at the left, and index 0.25, at the right:




Cube and Table

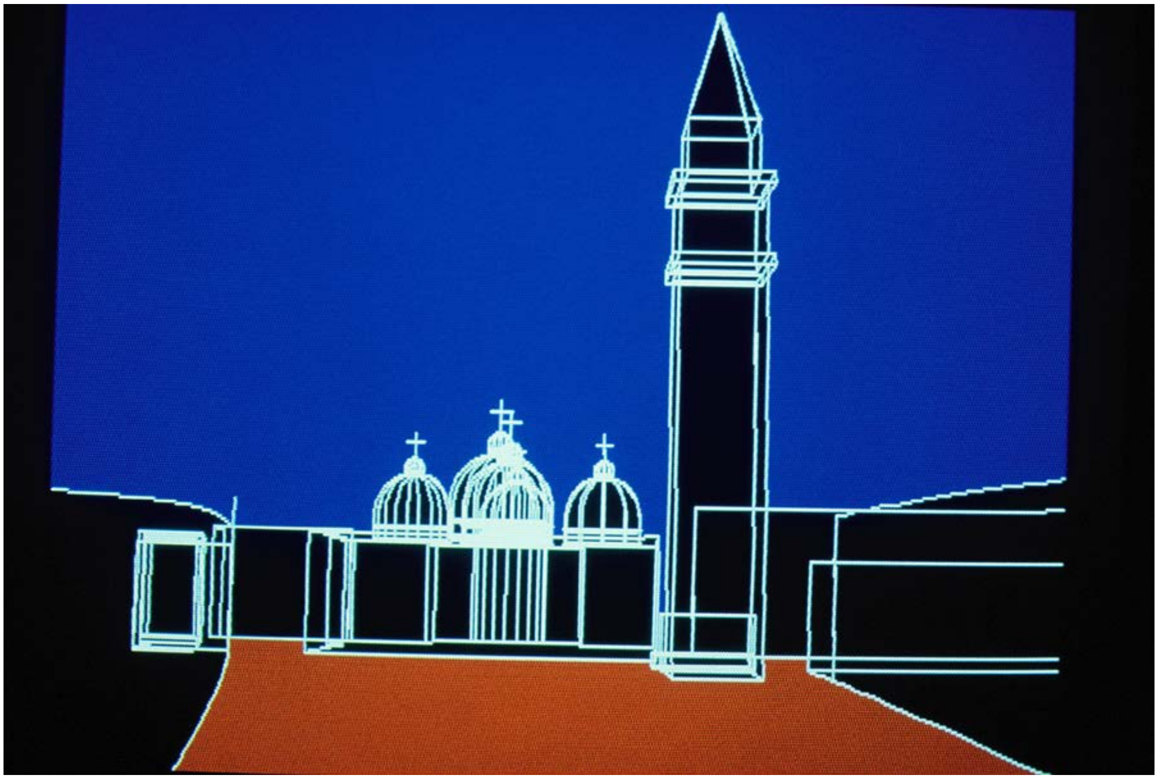
The curvature of lines in figures at the right with index non-zero tends to become more pronounced for viewpoints near the objects (usually large angles of view) or when the ratio of the distance from the viewpoint to the farthest point in the object to the distance to the closest one, measured along the line of sight, is high. Sets of similar figures were used in experiments by Margaret A. Hagen to evaluate the preferences of diverse groups of people with respect to several index i.

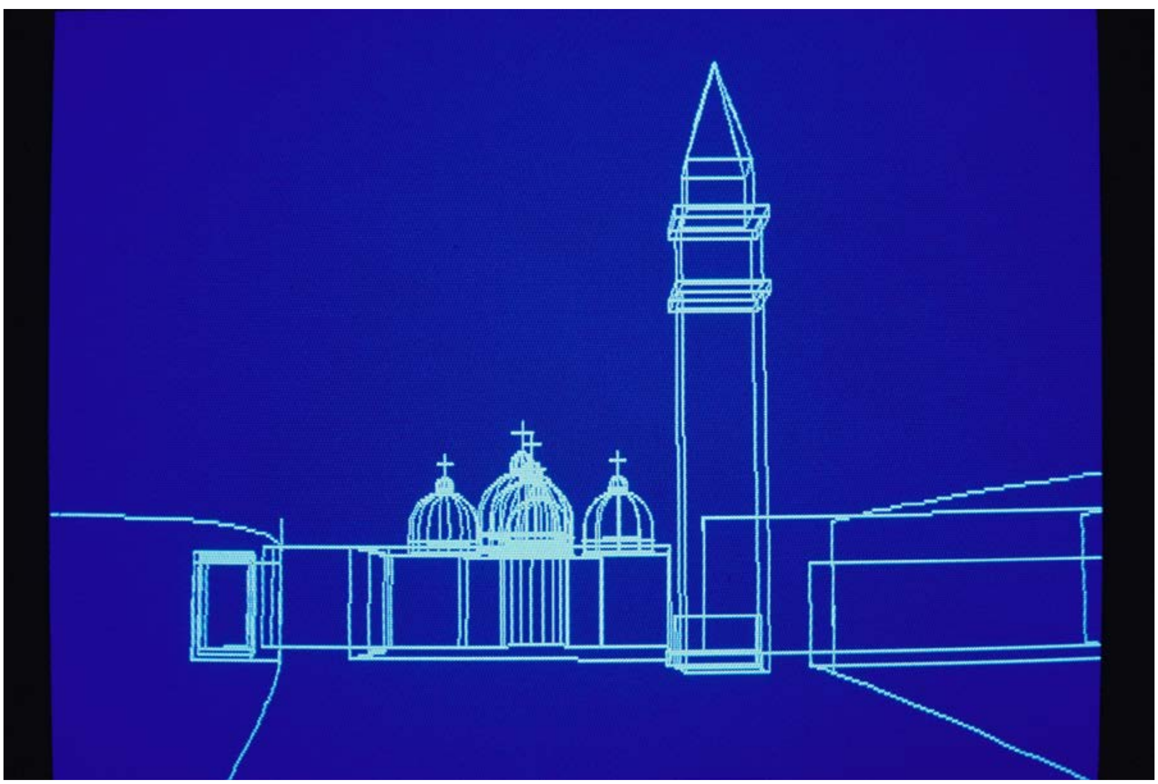
Now, the schematic pictures of a simulated urban complex (random example of building constructed in blocks of a city following an urban code):

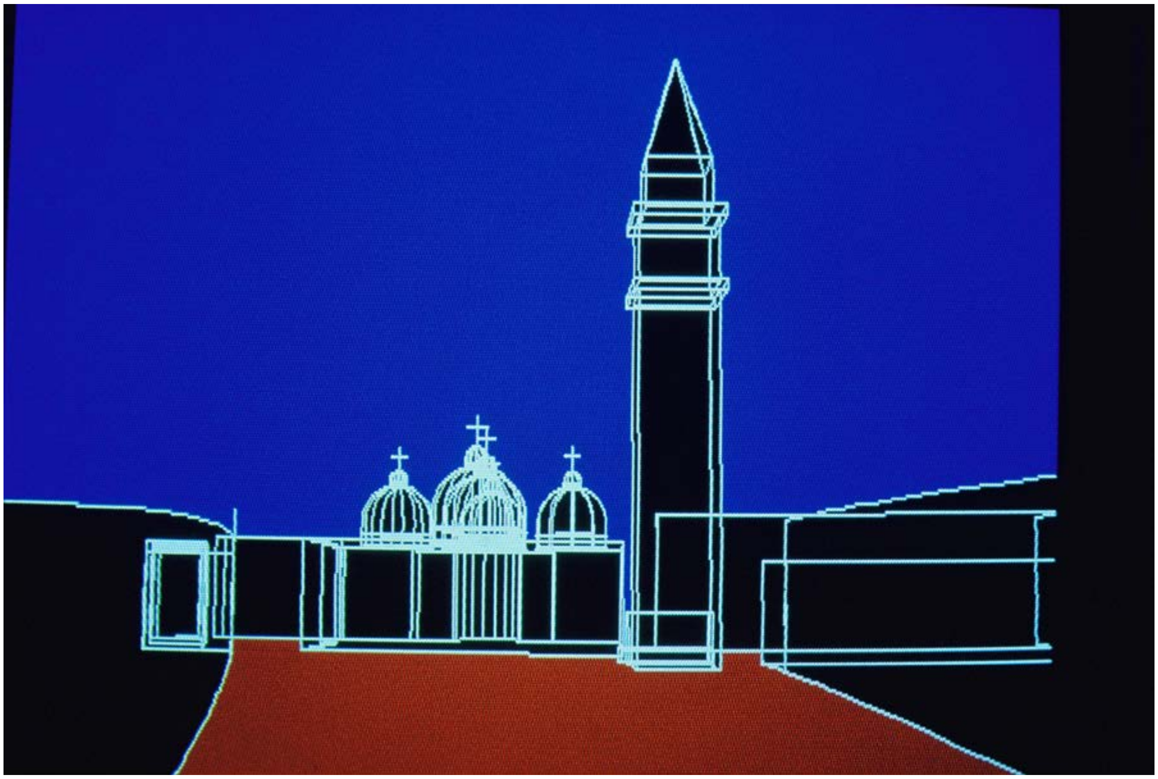


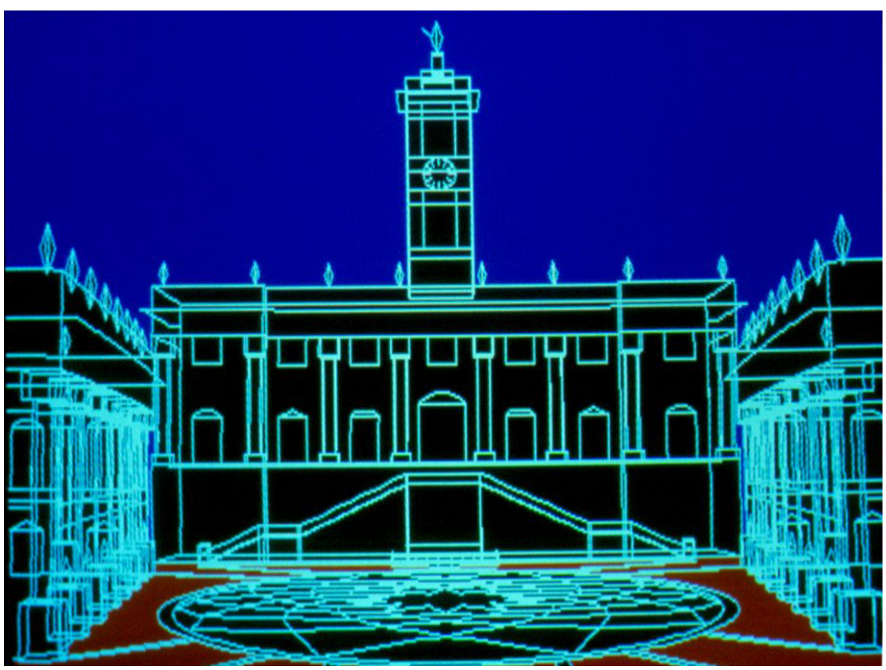
Urban Complex

And finally, St Mark's Square in Venice and Campidoglio in Rome
with i = 0.25:





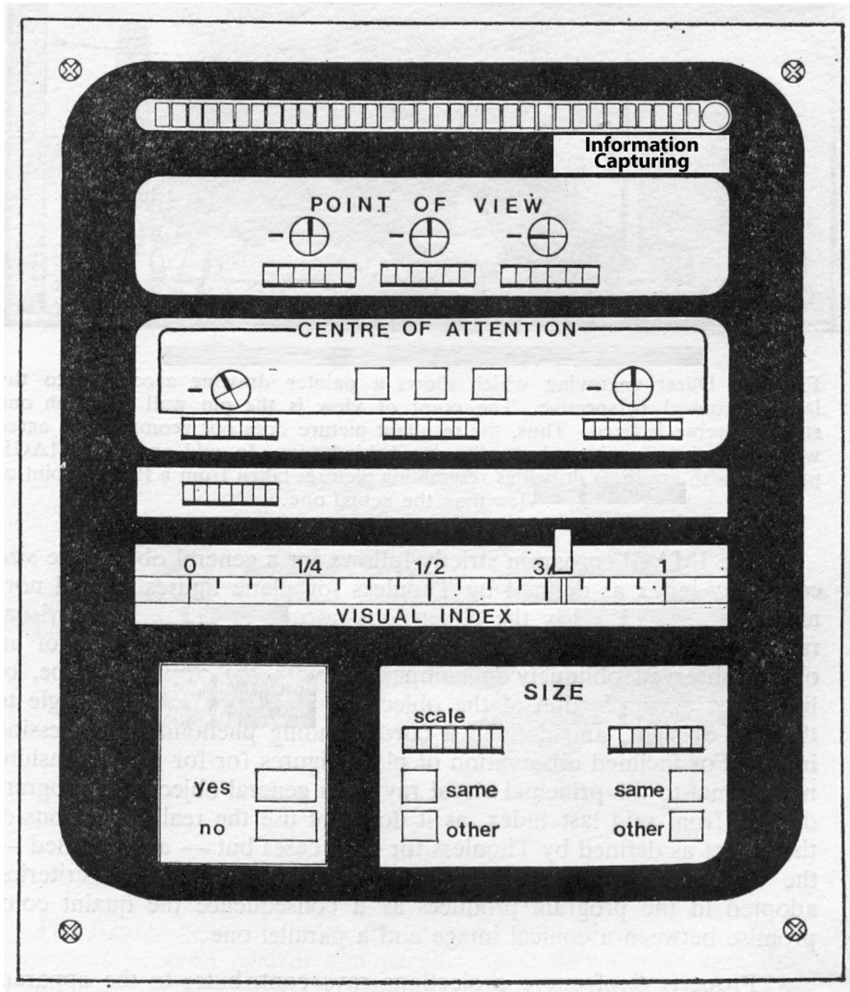




# Epilogue

I would like to say that the perspectives produced with the described procedure are a new type of plane representation of three-dimensional objects, only slightly different from the usual representation achieved with classical perspective for the normal distances and angles of observation. This is just a refinement that gives the user the freedom to choose his favorite image according to his individual taste. Although I began with Thouless's studies on perceptual size constancy, I do not claim complex psychological characteristics for the resulting images. Thouless's concept of size constancy is a controversial issue; therefore, I believe it is convenient to separate the use of the explained curved projection perspectives from psychological considerations. Vision has no significance for people unless they can relate it to things already present in their memory. For that reason I insist that the alternative introduced here of selecting different images by varying an index, is just a new tool that lengthens the list of existing techniques.

Finally, I imagine the possibility of a device that would generate images with different indexes taken directly from reality, turning a dial or control in the same way we now select special conditions on photographic and video cameras. Such a machine would need special sensors to evaluate the distances of all points of the objects and of the total scene, in order to reproduce the image corresponding to the selected index. In lectures twenty-five years ago, I spoke of such a machine. The probability of its construction was then uncertain, but this has changed due to the rapid and continuous improvement of computerized virtual reality and image manipulation techniques.




Thus, it will be possible for any person to obtain pictures with different indexes. The idea is to give people the possibility of being directors and not directed, to enhance and diversify the human condition.

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